An Optimized AHP-based methodology integrated in a Decision Support System for existing buildings safety

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Highlights

This paper shows one of the first applications of the Optimized Analytic Hierarchy processes (O-AHP) in the field of existing R.C. buildings safety. The work discusses the methodology phases to face the problem of degradation of existing buildings that can compromise usability and user safety. For this purpose, suitable Key Performance Indicators are obtained to support diagnostics. In addition, the indicators are set by exploiting data available in a Decision Support System (DSS). Finally, it is possible to obtain a building performance index by considering all the single surveyed pathologies.

Abstract

The large-scale management and monitoring of buildings is a complex issue for the heritage buildings holders. Decision Support Systems (DSS) are increasingly applied and studied in the field of building from the scientific and technical communities. Multiple-Criteria Decision Analysis (MCDA), such as Analytic Hierarchy Processes (AHP), are widely used in building performance analysis. On the other hand, AHP are difficult to apply in building field because of the complexity of the problem. This work presents an optimized approach to use AHP integrated in the DSS and applied in the field of the safety and monitoring of R.C. buildings.

Keywords

Multi-criteria decision methods, Building Pathologies, Decision Support Systems, Building Safety, Mathematical Optimization

1. INTRODUCTION

In the existent building management context at a regional scale, maintenance and monitoring activities perform a fundamental role. Pathologies related to the degradation of existing buildings can compromise usability and user safety [1]. Buildings holders need to monitor building criticalities by means of direct inspections and surveys to guarantee monitoring and safety.

In order to simplify such operations, Decision Support Systems (DSSs) are even more widespread to support diagnostics and building pathologies risk quantification. In such system the mathematical tool used to quantify building damages is crucial in order to obtain the useful information in the decision support [2].
A tool applied in many field is the Analytic Hierarchy Process (AHP) by which it is possible analyse different variables. The AHP is based on the decomposition of the problem into independent criteria: such an operation allows transforming a multidimensional scaling problem into a one-dimensional scaling problem. Such characteristics make the AHP particular effective to analyse problem in the field of existing building.

In particular, every criterion is analysed individually to identify the related Priority Vector by which it is possible to derive weight of each criterion and alternative [3]. The AHP identifies such Priority Vector from the matrix of judgements, that is a matrix generated by the comparison in pairs of each alternative or criterion [4][5]. In addition, the AHP quantifies the coherence of the assigned judgments and the reliability of obtained weights through the analysis of the consistency of the matrix of judgements, defined by Saaty [6] [7] [8]. However, AHP has some limits: mathematical and psychology studies determined that the human mind is limited to $7 \pm 2$ alternatives in simultaneous comparison and this limit prevents achievement of judgments consistency [9]. Consequently, the reliability of the results decreases as long as the number of criteria of comparison to obtain matrix of judgements becomes large. To this aim, it is particularly important to study methodologies that are able to improve the consistency of judgments [10] by transforming an inconsistent matrix into a consistent one [6][3]. The main objective of this work is to apply a methodology that overcomes the limits of the classical AHP. To this aim, the main steps of the classical AHP to determine the matrix of judgements are re-elaborated in the Optimized-AHP (O-AHP) procedure. In particular, it is possible to set up a mathematical optimization for the detection of a consistent matrix of judgments by defining a set of inequalities that substitute the judgment scale proposed by Saaty. In such a way, the Decision Maker (DM) can use just a range of judgments and focus on the main judgments instead of carrying out all of the pairwise comparisons. This approach can be useful in complex problems, when the decision maker does not have full system knowledge. From a mathematical point of view, a Mathematical Programming (MP) problem is formalized to deal with the inconsistency of the judgment matrix by minimizing the inconsistency performance index, subject to the inequalities imposed by the DM. Hence, the applied methodology allows overcoming the limit of the $7 \pm 2$ alternatives of the simultaneous comparison to reach consistent result. The O-AHP is applied to data obtained from DSS for monitoring on a territorial scale [11] in order to quantify the influence of building pathologies, with resulting detachment, fall of elements or collapse of the element itself, in building safety.
2. THE CLASSIC AHP METHOD AND THE APPLICABILITY LIMITS

The proposed O-AHP follows the footsteps of the well know standard AHP four steps [12] by operating on its mathematical formulas and by proposing the improvement of weights evaluation. To this aim the two main steps of standard AHP are described in this section: problem structuring and weights evaluation. Starting from a decision problem, the first step consists in structuring the problem according to a hierarchical scheme, in order to provide a detailed, simple, systematic and structured decomposition of the general problem into its basic components.

To this aim, the goal of the AHP is identified and the related criteria, sub-criteria and alternatives to reach the goal are determined (Fig. 1). This analysis of the problem allows studying each aspect of the decision problem individually. This approach is particular effective in the performance analysis of the building sector.

The second step of weight evaluation is the core of the method and provides the weights that are necessary for generating the ranking. In this phase the classical method presents the most evident limits in the application to the building field in which a large number of components are compared. More precisely, in this phase the matrix of judgments and the relative weights are determined.

Considering n ordered criteria of comparison (i.e., criteria, sub-criteria or alternatives in relation with criteria or sub-criteria), a \( n \times n \) judgments matrix \( A \) is defined (Fig. 2), where each upper diagonal element \( a_{ij} > 0 \) is generated by comparing the \( i^{th} \) element with the \( j^{th} \) one through the fundamental scale of absolute numbers.

This semantic scale is composed by verbal scales that are associated to numerical values (1, 3, 5, 7, 9) and compromises (2, 4, 6, 8) between such values (Fig. 3) [13].
Once a matrix of judgements is obtained, in the standard AHP, the weights are obtained by solving the following eigenvector problem:

$$A w = \lambda_{\text{max}} w,$$

where $w$ is the priority vector and $\lambda_{\text{max}}$ is the principal eigenvalue. Operatively, approximate formulation methods are used in order to calculate the weights from the judgments matrix [14].

The AHP method allows identifying the consistency, and consequently the reliability of the weights obtained, through a numerical index: the consistency index $CI$, defined by Saaty [12] as follows:

$$CI = \frac{\lambda_{\text{max}} - n}{n - 1}.$$

In order to provide a measure of the inconsistency that is independent of the matrix order, Saaty [8] proposed the Consistency Ratio (CR). This is obtained by considering the ratio between $CI$ and its expected value (Random Consistency Index - $RI$) determined over a large number of positive reciprocal matrices of order $n$, whose entries are randomly chosen in the set of values $n \in \{1, 2, \ldots, 10\}$

$$CR = CI / RI(n).$$
In this paper, in order to face a large number of alternatives, it is necessary to consider values of $RI(n)$ for $1 \leq n \leq 15$. Hence, among the different values of $RI$ proposed in the related literature, the values of Noble [15] are used (Fig. 4).

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RI$</td>
<td>0.00</td>
<td>0.49</td>
<td>0.82</td>
<td>1.03</td>
<td>1.16</td>
<td>1.25</td>
<td>1.31</td>
<td>1.36</td>
<td>1.39</td>
<td>1.42</td>
<td>1.44</td>
<td>1.46</td>
<td>1.48</td>
<td>1.49</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Random consistency index of Noble.

On the basis of several empirical studies, Saaty [8] concludes that the value of Consistency Ratio $CR < 0.10$ is acceptable. Such a test about $CR$ is crucial for establishing the reliability of assigned judgments and it is the parameter that mathematically determines the incoherence of the decision maker judgments. In the building sector, a large number of elements is used: this situation leads to unsatisfied consistency requirement of the comparison in most cases.

3. THE OPTIMIZED AHP PROCEDURE FOR WEIGHT CALCULATION

In order to overcome the consistency issue of AHP for problems with a large number of criteria or alternatives, the Optimized AHP (O-AHP) methodology for generating the judgement matrix $A$ is proposed. In particular, the presented method focuses on the second step of AHP, i.e., the optimized weights evaluation. The methodology can be described by considering three main phases: phase 1) rough ranking evaluation of the alternatives; phase 2) determination of a set of judgment ranges; phase 3) MP formulation of the O-AHP.

3.1 JUDGEMENT RANGES EVALUATION

In some cases, it is useful to preliminarily assume an approximate ranking of the criteria of comparison. To this aim, such criteria can be positioned starting from the first rows and columns of the judgement matrix $A$ in descending order of importance: this optional operation can simplify the subsequent steps.

Secondly, unlike the Saaty method, in which the DM performs the judgment by a crisp number, in the O-AHP method the decision maker expresses a judgement range, i.e., the range in which the judgement is assumed to belong. Judgement range setting is formalized through inequalities, by proposing two new semantic ranges of the O-AHP, inspired by the fundamental scale of absolute numbers of Saaty: the O-AHP semantic ranges of the lower bounds (Fig. 5) and the O-AHP semantic ranges of the upper bounds (Fig. 6).
The **Semantic Ranges** of the O-AHP are defined by following the footsteps of the fundamental scale of absolute numbers of Saaty, but unlike the Saaty method in which the DM has to express a precise judgment, in the O-AHP method the DM determines the JR and can omit judgments in cases of particularly difficult choice.

### 3.2 THE MATHEMATICAL FORMULATION OF THE O-AHP

The objective of the MP problem [16] is to determine the elements of the judgment matrix $A$ on the basis of the inequalities assigned in phase 2.

To this aim, the sets of pairs of sub-criteria that are subject to the judgement range inequalities are defined as follows:

- Lower bound constraint $K_{ij}^L$:
  - $a_{ij} \geq 1$: Equal or more importance of $i$ over $j$
  - $a_{ij} \geq 3$: More importance, even slightly, of $i$ over $j$
  - $a_{ij} \geq 5$: At least moderate importance of $i$ over $j$
  - $a_{ij} \geq 7$: At least strong importance of $i$ over $j$
  - $a_{ij} \geq 9$: At least very strong importance of $i$ over $j$
  - $1.5 - 4 - 6 - 8$: Intermediate values
  - $a_{ij} < 1/9, 1/8, ..., 1/2$: The reciprocal number the “≦” sign expresses an opposite judgment (becomes upper bound)

- Upper-bound constraint $K_{ij}^U$:
  - $a_{ij} \leq 3$: The importance of $i$ over $j$ does not exceed the "moderate importance"
  - $a_{ij} \leq 5$: The importance of $i$ over $j$ does not exceed the "strong importance"
  - $a_{ij} \leq 7$: The importance of $i$ over $j$ does not exceed the "very strong importance"
  - $a_{ij} \leq 9$: The importance of $i$ over $j$ does not exceed the Max importance
  - $1.5 - 4 - 6 - 8$: Intermediate values
  - $a_{ij} > (1/9, 1/8, ..., 1/2)$: The reciprocal number the greater-than sign expresses an opposite judgment (becomes lower bound)

Hence, it is possible to specify the entries of $A$ by the following set $\Gamma(A)$ of mathematical constraints:

$\Gamma(A)$:

$$a_{ij} = 1 \quad \text{for } i,j = 1,...,n \text{ with } i=j \quad (4a)$$

$$1/9 < a_{ij} < 9 \quad \text{for } i,j = 1,...,n \text{ with } i>j \quad (4b)$$
where the value of $CI$ is calculated according to (2).

The optimized matrix of judgments $A'$ and $a'_{ij}$ is obtained by solving the MP (5a-b).

The matrix consistency and in particular the relative optimized weights $w'$ are obtained by solving the problem (1) as the classical method.

The methodological simplifications of the proposed O-AHP are the following:

1. the judgments necessary to obtain the entries of $A$ are not determined by single values of comparison but they are expressed by a set of ranges;
2. the consistency of matrix $A$ is not obtained through a trial and error method as in the standard $AHP$, but it is optimized by the solution of the MP problem, improving the coherence of the judgments.

These features make the O-AHP method effective even when the number of comparisons becomes greater than 10.

4. APPLICATION OF THE O-AHP IN THE DSS

In this section the O-AHP methodology [16] is applied to quantify safety by comprising the identification of the building pathologies through the structure of the problem and the identification of criteria and alternatives weights.

In particular, the criteria and the alternatives are set by considering the data available in the DSS [11].

The considered criteria are: i) pathology severity; ii) pathology extension compared to the element dimension; iii) and criticality height above walking surface.

Fig. 7 schematizes the decomposition of the problem into criteria and alternative according to a hierarchical scheme.

The weights evaluation is performed starting from the alternatives of the criterion Severity of pathology. A rough ranking evaluation is carried out and the JR$s$ are evaluated by use the fundamental scales of O-AHP. (Fig. 5 and 6).
In particular the upper bound $K_{ij}^U$ and the lower bound $K_{ij}^L$ are defined. We proceed by imposing constraints for the matrix $A'_1$, (Fig. 8), and the uncertain judgments are omitted. Consequently, JRs are determined and shown schematically in Fig. 9.

In the next phase the problem is formalized through the mathematical constraints expressed in (4a) (4b) (4c) (4d) (4e). The constraints $K_{ij}^U$ and $K_{ij}^L$ are shown in Fig. 9.

Fig. 8 shows the solution of the optimization problem (5a-b) that provides the optimized matrix of judgment $A'_1$. In addition, by solving the eigenvalues problem (1) [13] the optimized and normalized weights $w'_1$ are obtained. The optimized matrix $A'_1$ respects the consistency requirement of $CR' = 0.0174 < 0.1$ and values of matrix $A'_1$ result coherent according to the Saaty theory.

![Figure 7. Hierarchical structuring of the problem.](image)

![Figure 8. Matrix of judgement $A'_1$ optimized for the alternatives of the criterion Severity of pathology.](image)
The described procedure is repeated to obtain the weights $w'_2$ of the alternatives of criterion pathology extension and the weights $w'_3$ of the alternatives of criterion criticality height above walking surface. Finally, weight of criteria $\lambda_1$, $\lambda_2$, $\lambda_3$ with respect to the objective are determined.

Fig. 10 shows all the weights obtained by the O-AHP application.

### Table: Severity of pathology

<table>
<thead>
<tr>
<th>Severity of pathology</th>
<th>$w'_2$</th>
<th>Pathology extension</th>
<th>$w'_3$</th>
<th>Criticality height above walking surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cracks active, crack size of column beam or Walls, leaning out of vertical</td>
<td>10.6</td>
<td>100%</td>
<td>7.7</td>
<td>More than 4m</td>
</tr>
<tr>
<td>Brocken brick blocks and heavy finishing falling down</td>
<td>9.3</td>
<td></td>
<td>4.4</td>
<td>Between 4m and 6m</td>
</tr>
<tr>
<td>Heavy explosion of concrete cover and rebar oxidation</td>
<td>7.1</td>
<td></td>
<td>4.1</td>
<td>Between 6m and 8m</td>
</tr>
<tr>
<td>Pathology cause of section reduction of a beam</td>
<td>6.7</td>
<td></td>
<td>4.0</td>
<td>Between 8m and 10m</td>
</tr>
<tr>
<td>Loss of adhesion and detachment of finishing (also spalling)</td>
<td>4.6</td>
<td></td>
<td>1.0</td>
<td>less than 4m</td>
</tr>
<tr>
<td>Excessive deflection, deformations or displacements beam wall column</td>
<td>3.7</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Heavy foundation problem</td>
<td>3.8</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Medium cracks or steel reinforcement beam column or non structural steel element</td>
<td>3.8</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Component deformation beam column or wall</td>
<td>3.8</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Generic thin cracks of defects at junctions</td>
<td>2.0</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Decoy phenomena superficial</td>
<td>0.8</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Wood or steel pathology (not heavy)</td>
<td>1.0</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Medium settlement or differential settlement</td>
<td>1.0</td>
<td></td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The obtained weights quantify the safety compromise identification due to the building pathologies reported by the DSS. Let us consider a structure decomposed in a set $F\{f=1,...,N_f\}$ of components
classes (shear wall, column, beam, etc.) and a set $E=\{e=1,\ldots,N_E\}$ of possible components. Subsequently, a report is generated on the basis of the visual analysis for each building’s pathology: every report is composed of a photographic survey and additional structured information, organized in the set of alternatives.

For each reported pathology, the DSS can obtain a numerical value for each report that quantifies the building safety compromise ($C_{re}$) by exploiting the following formula:

$$C_{re} = \lambda_1 w_1 + \lambda_2 w_2 + \lambda_3 w_3$$  \hspace{1cm} (6)$$

where $C_{re}$ is the index associated to the $e$-th component and ranges from 0 to 10, $\lambda_1$, $\lambda_2$, $\lambda_3$ are the weights of the criteria, and $w_1$, $w_2$, $w_3$ are the weights associated to the alternatives according to Fig. 10.

To provide an example a report composed by “Cracks active, wide of column or Walls leaning out of vertical”, extended for 100% of the element and positioned at a height greater than 4 meters above walking surface results:

$$C_{re} = 0.75*10 + 0.29*10 + 0.14*10 = 10.$$  

$C_{re}$ is the maximum criticality value obtainable from the reports for the $e$-th component.

The synthetic index that quantifies the deterioration of the $f$-th class of components, named component condition rating of class $f$ ($CC_{rf}$), is calculated by aggregating the values of $C_{re}$ representing the criticality condition rating of component $e$ belonging to the same $f$-th class (obtained by eq. (6)). More precisely, the value of $CC_{rf}$ is computed as follows:

$$CC_{rf} = \frac{\sum_{e=1}^{N_E} C_{re}}{\gamma_f * S_{tot}},$$  \hspace{1cm} (7)$$

where $S_{tot}$ is the building total area and $\gamma_f$ is the approximated evaluation of the number of the considered components per square meter [17]. The sum in eq. (7) includes the subset of components $e$ related to class $f$. Finally, the degree of deterioration of the whole structure ($BC_{r}$) is calculated by aggregating the values of $C_{re}$ associated to all the elements belonging to the same building through the following expression:

$$BC_{r} = \frac{\sum_{e=1}^{N_E} C_{re}}{\gamma_p * S_{tot}},$$  \hspace{1cm} (8)$$

where $S_{tot}$ is the building total area and $\gamma_p$ is the approximated evaluation of the number of the components per square meter, evaluated for the specific R.C. building typology [17].
5. CONCLUSIONS

The paper presents a DSS that supports the buildings performance analysis by using mathematical formulations and algorithms for the performance index quantification. In such system the used mathematical methodology is crucial to identify criteria and weighing system to quantify building damages. This work discusses the main procedural phases of the methodology called Optimized AHP [16] and shows an application in the building field. This methodology allows applying the AHP methodology even if a large number of criteria and comparisons are necessary and the consistency is not easy to be obtained.

Moreover, from the mathematical point of view, the reliability and coherence of the resulting weights evaluated by the judgement matrix are critical issues when the number of alternatives increases and the standard AHP is applied. The proposed methodology overcomes the standard AHP drawbacks by revising the AHP weights evaluation procedure. Firstly, the exact judgement assignments are replaced by judgment ranges. Secondly, the entries of the judgement matrix are provided by a MP formulation that minimizes the inconsistency of the matrix subject to the constraints imposed on the weights. In particular, the O-AHP methodology is used to quantify the safety compromises due to the reinforced concrete building pathologies also by employing the parameters obtained by the DSS. In this context a large number of alternatives have to be considered and the standard AHP provides an inconsistent matrix. On the contrary, the application of the O-AHP allows obtaining consistent results.

Future research will apply the O-AHP to analyse the safety performance in relation with the seismic behaviour by improving the number of pathologies considered in the analysis.

6. REFERENCES


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